

WRITTEN CALCULATION PROGRESSION

SINGLE-AGE CLASSES

ESSENTIALMATHS Written Calculation Guidance

This document maps the **ESSENTIALMATHS** pathway to the required written formal calculation methods as outlined in the National Curriculum (2013) [Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division.](#)

The **ESSENTIALMATHS** Written Calculation Progression links the key concrete experiences with pictorial and abstract representations (written symbolic and spoken). This supports pupils to move with confidence and deep conceptual understanding through each strand of calculation.

The Importance of Mental Mathematics

While this progression focuses on written calculation in mathematics, **ESSENTIALMATHS** recognises the importance of mental strategies and known facts that form the basis of all calculations. A range of mental strategies are developed throughout **ESSENTIALMATHS**. Pupils are provided with frequent opportunities to compare and evaluate different calculation strategies. This helps them develop an understanding that efficiency is personal and based on the numbers involved.

The **ESSENTIALMATHS** Rehearsal and Reasoning Sheets have been designed to be used in addition to the handouts and resources contained within the **ESSENTIALMATHS** sequences and steps.

Concrete, Pictorial and Abstract

Concrete manipulatives

Concrete manipulatives are objects that can be touched and moved by pupils to introduce, explore or reinforce a mathematical concept. They provide a vehicle to help pupils make sense of complex, symbolic and abstract ideas through exploration and manipulation. Furthermore, they support the development of internal models and help build stronger memory pathways.

Pictorial (including jottings)

The act of translating the concrete experience into a pictorial representation helps focus attention on what has happened and why. This supports deeper understanding and a stronger imprint on memory. Pictorial representations are more malleable than concrete resources and, once understanding is secured, allow exploration of complex problems that may be challenging to reproduce with manipulatives.

Abstract - Written

The aim, within this policy, is for compacted forms of notation. These have developed through the history of mathematics. Explicit individual steps in procedure are hidden or they have been shortcut. The informal and expanded methods expose all the intermediate steps, replicating thought processes more closely and support understanding prior to compaction.

Abstract - Spoken

Learning to use the correct mathematical vocabulary is vital for the development of mathematical proficiency. The ability to articulate accurately allows pupils to communicate and build meaning. Ideas become more permanent. This can be scaffolded effectively using speaking frames.

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Addition		Subtraction	
2LS15	<p>Step 3: Adding two 2-digit numbers using a written method with no regrouping</p> <p>Step 4: Adding two 2-digit numbers using a written method with regrouping of ones</p>	2LS17	<p>Step 4: Subtracting a 2-digit number from a 2-digit number with no regrouping</p> <p>Step 5: Subtracting a 2-digit number from a 2-digit number with regrouping</p>
3LS8	<p>Step 2: Formal written method with no regrouping (exchange)</p> <p>Step 3: Formal written method with regrouping of ones</p> <p>Step 4: Regrouping tens and ones</p>	3LS9	<p>Step 1: Formal written method with no regrouping (exchange)</p> <p>Step 2: Subtraction – regrouping (exchange) tens into ones only</p> <p>Step 3: Subtraction – regrouping hundreds into tens only</p> <p>Step 4: Subtraction – regrouping hundreds and tens</p>
4LS4	<p>Step 1: Formal addition method with no regrouping (thousands, hundreds, tens and ones) *</p> <p>Step 2: Formal addition method with regrouping in hundreds, tens and ones *</p> <p>Step 3: Formal addition method with regrouping in hundreds, tens and ones causing a further thousand *</p>	4LS4	<p>Step 5: Revisiting formal written subtraction (decomposition)*</p> <p>Step 6: Formal written subtraction with regrouping of thousands (decomposition)*</p>
5LS10	<p>Step 2: Column addition*</p>	5LS10	<p>Step 3: Column subtraction*</p>

*indicates that the step is not explicitly exemplified within this progression as it is a revisit or extension of a previously exemplified step

Multiplication		Division	
3LS26	<p>Step 3: Introducing short multiplication with no regrouping</p> <p>Step 4: Short multiplication with regrouping of tens and ones only</p> <p>Step 5: Short multiplication with regrouping of tens and ones</p>	3LS30	<p>Step 2: Introducing long division (sharing structure)</p> <p>Step 3: Long division of tens and ones with no regrouping</p> <p>Step 4: Long division of tens and ones with regrouping</p>
4LS24	<p>Step 4: Short multiplication with no regrouping *</p> <p>Step 5: Short multiplication; with regrouping in one column</p>	4LS25	<p>Step 2: Long division with regrouping hundreds into tens</p> <p>Step 4: Short division</p>
5LS11	<p>Step 1: Revision of short written method for a 2 or 3-digit number by a 1-digit number extending to a 4-digit number by a 1-digit number *</p> <p>Step 2: Using grid method and expanded vertical method to solve a 2-digit number by a 2-digit number multiplication</p> <p>Step 3: Long multiplication of a 2-digit number by a 2-digit number</p>	5LS12	<p>Step 2: Sharing and grouping</p> <p>Step 3: Short division for numbers up to 4-digits</p> <p>Step 4: Expressing quotients with fractions</p> <p>Step 5: Expressing quotients with decimals</p>
6LS6	<p>Step 5: Long multiplication involving numbers with up to 2 decimal places multiplied by a 1-digit number</p> <p>Step 6: Application of short multiplication</p>	6LS17	<p>Step 2: Long division for numbers up to 4 digits</p> <p>Step 4: Expressing quotients with fractions</p> <p>Step 5: Expressing quotients with decimals</p>

*indicates that the step is not explicitly exemplified within this progression as it is a revisit or extension of a previously exemplified step

ADDITION PROGRESSION

Year 2

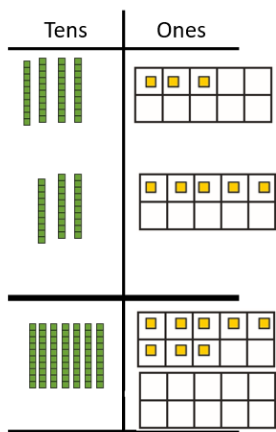
NC Statement:

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

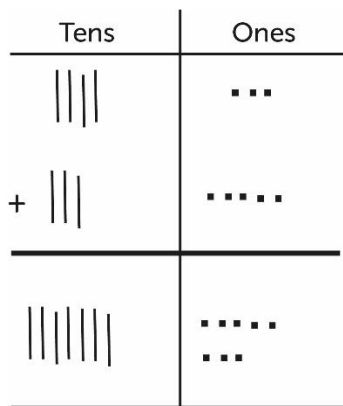
- a two-digit number and ones
- a two-digit number and tens
- two, two-digit numbers

2LS15 Step 3: Adding two 2-digit numbers using a written method with no regrouping

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} 40 \quad 3 \\ + 30 \quad 5 \\ \hline 70 \quad 8 \end{array}$$

$$43 + 35 = 78$$

Abstract - Speaking frame

The sum of ... ones and ... ones is ... ones.
 The sum of ... tens and ... tens is ... tens.
 So, ... + ... is equal to ... tens and ... ones, which is ...

Notes:

Using embedded tens frame supports pupils to organise ones in preparation for regrouping.

Addition

Year 2

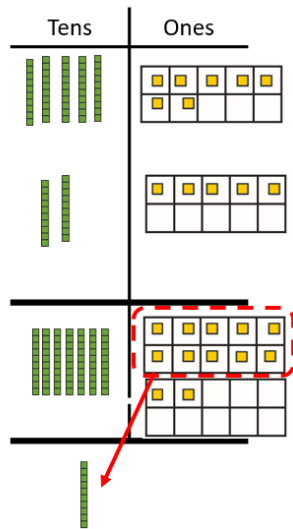
NC Statement:

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

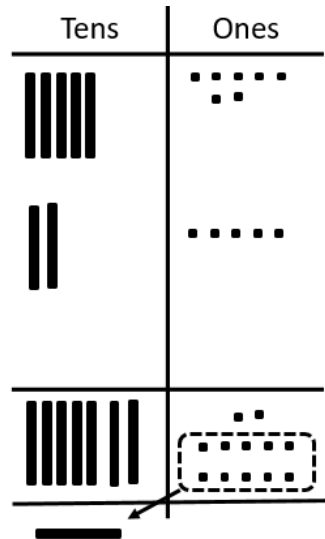
- a two-digit number and ones
- a two-digit number and tens
- two, two-digit numbers

2LS15 Step 4: Adding two 2-digit numbers using a written method with regrouping of ones

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} 50 + 7 \\ + 20 + 5 \\ \hline 80 + 12 \\ \hline 10 \quad 2 \end{array}$$

$$57 + 25 = 82$$

Abstract - Speaking frame

The sum of ... ones and ... ones is ... ones.
 This is regrouped into ... ten and ... ones.
 The sum of ... tens and ... tens is ... tens.
 So, ... + ... is equal to ... tens and ... ones, which is ...

Notes:

Pupils should be encouraged to estimate first and check their answer using a mental method.

Using embedded tens frame supports pupils to rapidly see the regroup and to keep their jottings organised.

Addition

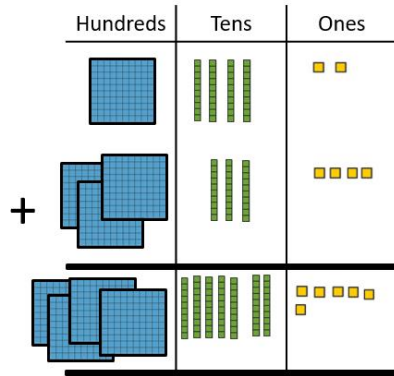
Year 3

NC Statement:

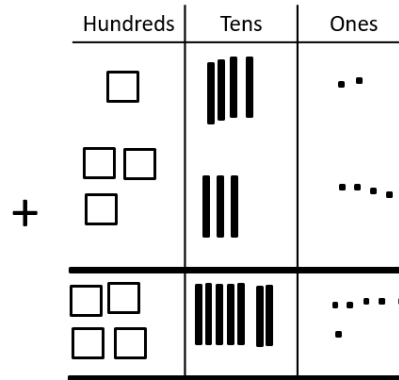
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

3LS8 Step 2: Formal written method with no regrouping (exchange)

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} 142 \\ + 334 \\ \hline 476 \end{array}$$

$$142 + 334 = 476$$

Abstract - Speaking frame

The sum of ... ones and ... ones is ... ones.
 The sum of ... tens and ... tens is ...tens.
 The sum of ... hundreds and ... hundreds is ... hundreds.
 So, ... + ... is equal to ... hundreds, ... tens and ... ones,
 which is

Notes:

3LS8 Step 2 revisits the formal written method, first encountered in Year 2, with no regrouping but introduces hundreds.

Pupils should be encouraged to estimate first and check their answer using a mental method.

Addition

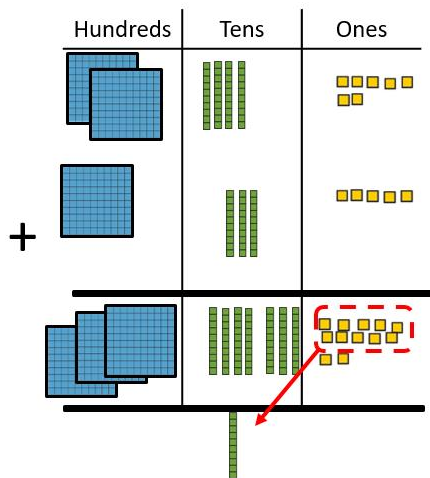
Year 3

NC Statement:

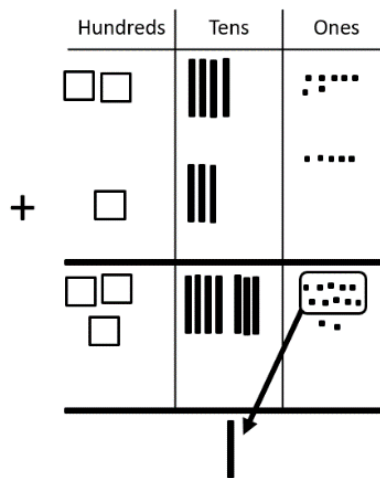
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

3LS8 Step 3: Formal written addition with regrouping of ones

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} 247 \\ + 135 \\ \hline 382 \\ \hline 1 \end{array}$$

$$247 + 135 = 382$$

Abstract - Speaking frame

The sum of ... ones and ... ones is ... ones.
 The sum of ... tens and ... tens is ...tens.
 The sum of ... hundreds and ... hundreds is ... hundreds.
 So, ... + ... is equal to ... hundreds, ... tens and ... ones,
 which is

Notes:

The focus is on regrouping of ones.

Pupils should be encouraged to estimate first and check their answer using a mental method.

Addition

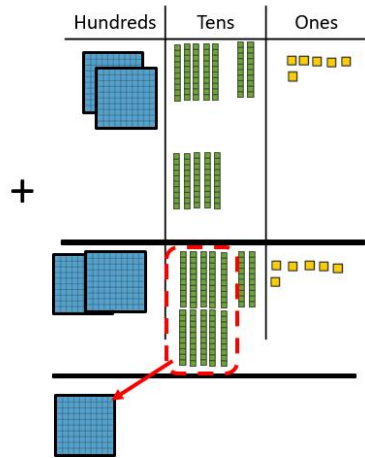
Year 3

NC Statement:

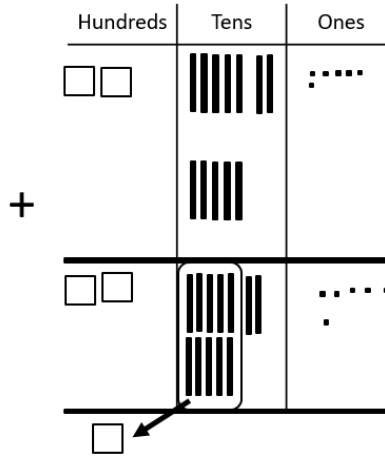
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

3LS8 Step 4: Regrouping tens and ones

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} 276 \\ + 50 \\ \hline 326 \\ \hline 1 \end{array}$$

$$276 + 50 = 326$$

Abstract - Speaking frame

The sum of ... ones and ... ones is ... ones.
 This is regrouped into ... tens and ... ones.
 The sum of ... tens and ... tens is ...tens.
 The sum of ... hundreds and ... hundreds is ... hundreds.
 So, ... + ... is equal to ... hundreds, ... tens and ... ones, which is

Notes:

The focus is on regrouping of tens.

Pupils should be encouraged to estimate first and check their answer using a mental method.

Addition

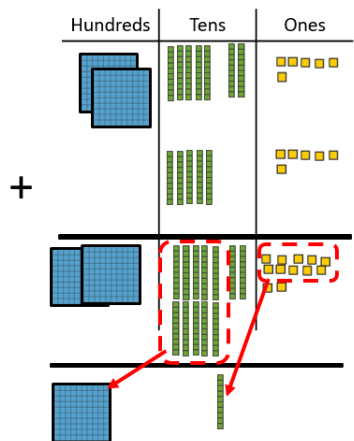
Year 3

NC Statement:

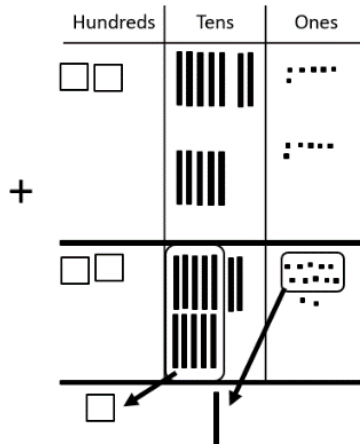
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

3LS8 Step 4: Regrouping tens and ones

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} 276 \\ + 56 \\ \hline 332 \\ \hline 11 \end{array}$$

$276 + 56 = 332$

Abstract - Speaking frame

The sum of ... ones and ... ones is ... ones.
 This is regrouped into ... tens and ... ones.
 The sum of ... tens and ... tens is ...tens.
 This is regrouped into ... hundreds and ... tens.
 The sum of ... hundreds and ... hundreds is ... hundreds.
 So, ... + ... is equal to ... hundreds, ... tens and ... ones, which is ...

Notes:

Pupils should be encouraged to estimate first and check their answer using a mental method.

Once pupils have fully understood and rehearsed regrouping within formal column addition of 3-digit numbers, this learning continues to be rehearsed and applied throughout Years 4, 5 and 6, including to 4-digit numbers, larger numbers, decimal numbers, money and measures.

Addition

SUBTRACTION PROGRESSION

Year 2

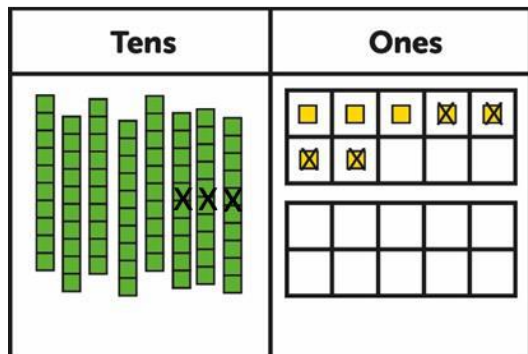
NC Statement:

add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

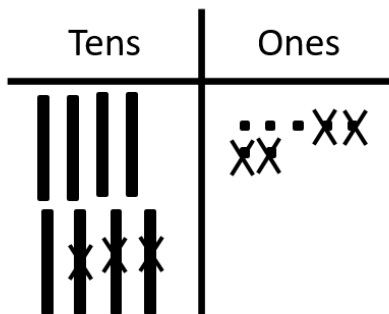
- a two-digit number and ones
- a two-digit number and tens
- two, two-digit numbers.

2LS17 Step 4: Subtracting a 2-digit number from a 2-digit number with no regrouping

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} 80 + 7 \\ - 30 + 4 \\ \hline 50 + 3 \end{array}$$

$$87 - 34 = 53$$

Abstract - Speaking frame

... ones take away ... ones leaves ... ones.
 ... tens take away ... tens leaves ... tens.
 So, ... - ... is equal to ... tens and ... ones, which is

Notes:

Pupils should be encouraged to estimate first and check their answer using a mental method.

Subtraction

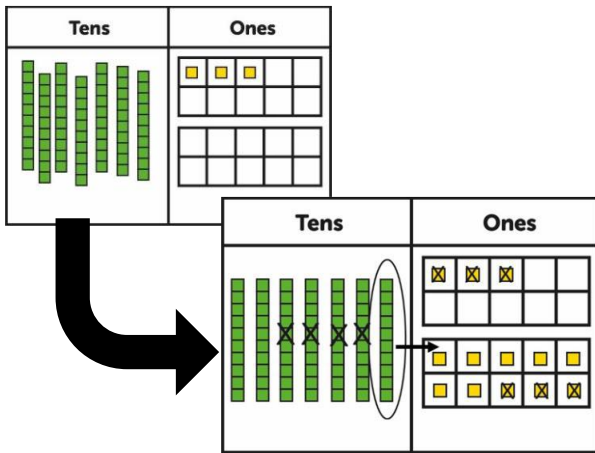
Year 2

NC Statement:

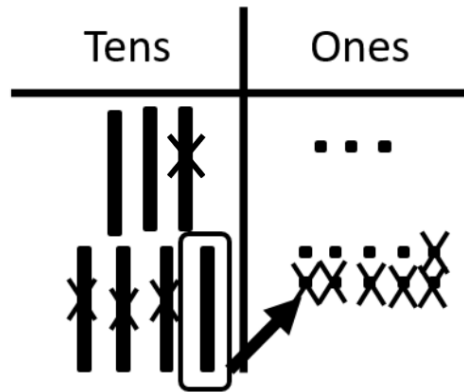
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
- a two-digit number and ones
 - a two-digit number and tens
 - two, two-digit numbers.

2LS17 Step 5: Subtracting a 2-digit number from a 2-digit number with regrouping

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r}
 60 \quad 13 \\
 70 + 3 \\
 - 40 + 6 \\
 \hline
 20 + 7 \\
 \hline
 \end{array}$$

$$73 - 46 = 27$$

Abstract - Speaking frame

I can see that there aren't enough ones for me to take away ... ones without regrouping.
 Regroup one ten into ten ones.
 There are now ... tens and ... ones.
 ... ones take away ... ones leaves ... ones.
 ... tens take away ... tens leaves ... tens.
 So, ... - ... is equal to ... tens and ... ones, which is

Notes:

Using embedded tens frame supports pupils to regroup accurately and to keep their jottings organised.

Speaking frame note: "I can see that there aren't enough ones for me to take away 6 ones without regrouping. Regroup one ten into ten ones. There are now 6 tens and 13 ones."

Subtraction

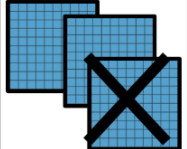
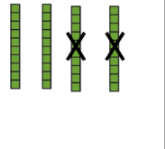

Year 3

NC Statement:

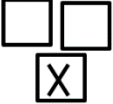


Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

3LS9 Step 1: Formal written method with no regrouping (exchange)

Concrete

Hundreds	Tens	Ones
		

Pictorial

Hundreds	Tens	Ones
		

Abstract - Written symbolic

$$\begin{array}{r} 345 \\ - 124 \\ \hline 221 \end{array}$$

$$345 - 124 = 221$$

Abstract - Speaking frame

... ones take away ... ones leaves ... ones.
 ... tens take away ... tens leaves ... tens.
 ... hundreds take away ... hundreds leaves ... hundreds.
 So, ... - ... is equal to ... hundreds, ... tens and ... ones, which is ...

Notes:

Pupils should be encouraged to estimate first and check their answer using a mental method.

Subtraction

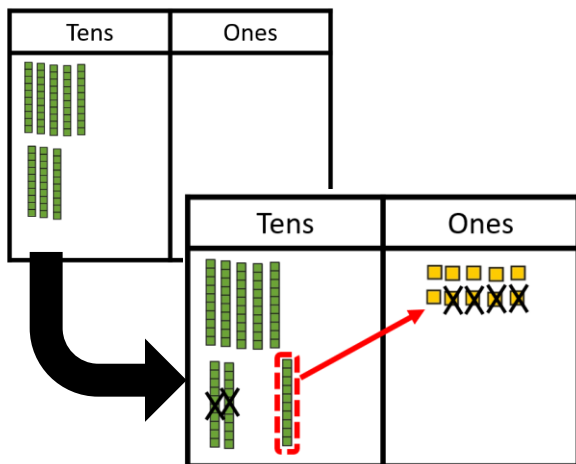
Year 3

NC Statement:

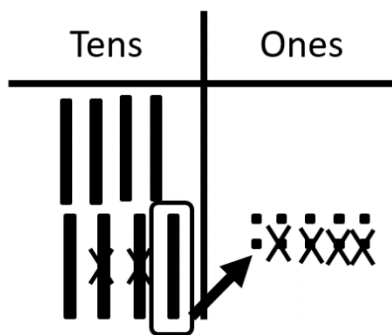
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

3LS9 Step 2: Subtraction – regrouping (exchange) tens into ones only

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} \overset{7}{\cancel{8}} \overset{10}{0} \\ - 24 \\ \hline 56 \end{array}$$

$$80 - 24 = 56$$

Abstract - Speaking frame

I can see that there aren't enough ones for me to take away ... ones without regrouping.
 Regroup one ten into ten ones.
 There are now ... tens and ... ones.
 ... ones take away ... ones leaves ... ones.
 ... tens take away ... tens leaves ... tens.
 So, ... - ... is equal to ... tens and ... ones, which is

Notes:

It is important that pupils understand that 80 has been regrouped into 70 and 10. If pupils struggle with the compact notation refer to 2LS17 Step 5 for the expanded method.

Speaking frame note:

"I can see that there aren't enough ones for me to take away 4 ones without regrouping. Regroup one ten into ten ones. There are now ten ones and zero ones. 10 ones take away 4 ones leaves six ones. 7 tens take away 2 tens leaves 5 tens. So, 80 - 24 is equal to 5 tens and 6 ones, which is 56."

Subtraction

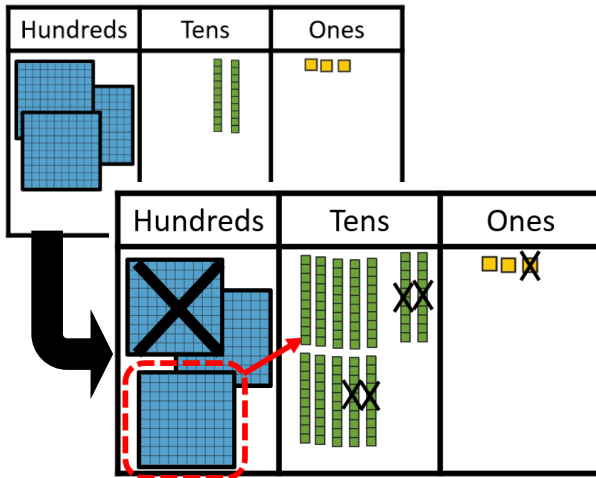
Year 3

NC Statement:

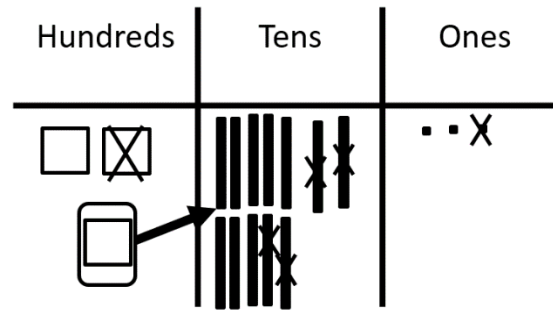
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

3LS9 Step 3: Subtraction – regrouping hundreds into tens only

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} \cancel{2}^2 \cancel{2}^1 3 \\ - 141 \\ \hline 182 \end{array}$$

$$323 - 141 = 182$$

Abstract - Speaking frame

... ones take away ... ones leaves ... ones.
 I can see that there aren't enough tens for me to take away ... tens without regrouping.
 Regroup one hundred into ten tens.
 There are now ... hundreds and ... tens.
 ... tens take away ... tens leaves ... tens.
 ... hundreds take away ... hundreds leaves ... hundreds
 So, ... - ... is equal to ... hundreds, ... tens and ... ones, which is

Notes:

It is important that pupils start to identify where regrouping is necessary. Ensure that pupils are confident that the minuend may have been regrouped but it is still of equal value prior to subtraction.

Subtraction

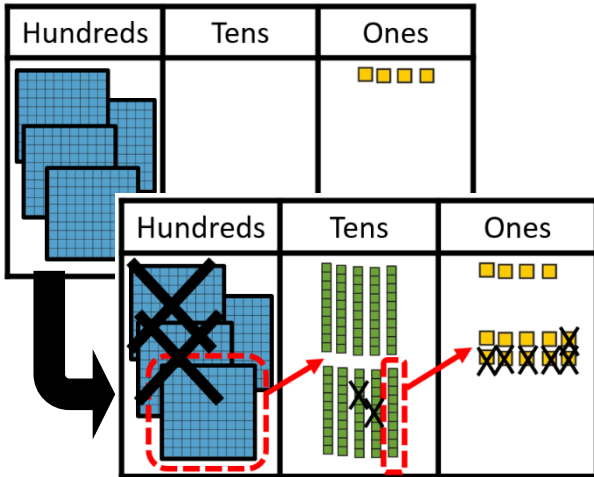
Year 3

NC Statement:

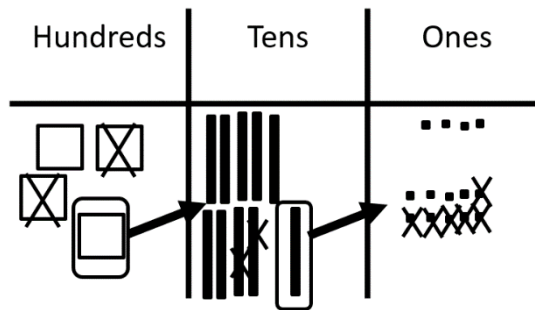
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

3LS9 Step 4: Subtraction - regrouping hundreds and tens

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r}
 \overset{3}{\cancel{4}} \overset{19}{\cancel{0}} \overset{1}{4} \\
 - 226 \\
 \hline
 178
 \end{array}$$

$$404 - 226 = 178$$

Abstract - Speaking frame

I will need to regroup...

- one hundred into ten tens. I now have ... hundreds and ... tens.
- one ten into ten ones. I now have ... tens and ... ones.

Notes:

Speaking frame hint: This is not a complete speaking frame. It is structured to support pupils with the language of regroup only.

Once pupils have fully understood and rehearsed regrouping within formal subtraction, this learning continues to be rehearsed and applied throughout Years 4, 5 and 6, including to multi-digit, decimal numbers, money and measures.

Subtraction

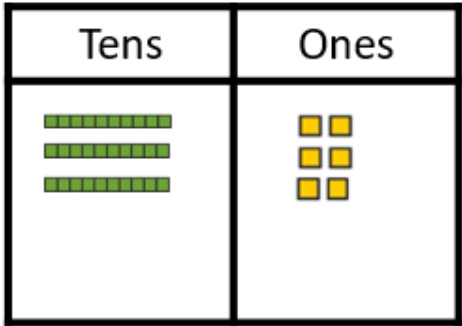
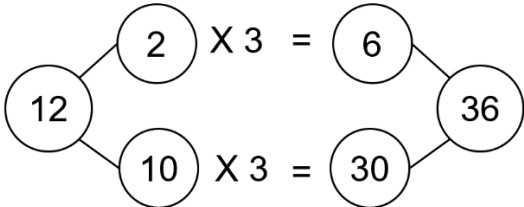
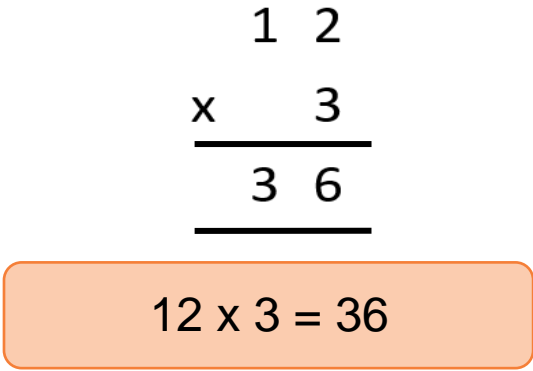
MULTIPLICATION PROGRESSION

Year 3

NC Statement:

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

3LS26 Step 3: Introducing short multiplication with no regrouping

Concrete	Pictorial - Jottings	Abstract - Written symbolic
		
<h4>Abstract - Speaking frame</h4> <div style="border: 2px solid orange; border-radius: 15px; padding: 10px; text-align: center;"> <p>... groups of ... ones is ... ones. ... groups of ... tens is ... tens. ... tens added to ... ones is ... The product of ... and ... is ...</p> </div>		<h4>Notes:</h4> <p>Pupils have already met the distributive law (3LS18) and rehearsed multiplying by ten (3LS25). The focus of this step is supporting pupils in making the connection between informal distributive approach and the formal layout.</p> <p>Speaking frame note: <i>“3 groups of 2 ones is 6 ones. 3 groups of 1 ten is 3 tens. 3 tens added 6 ones is 36. The product of 12 and 3 is 36.”</i></p>

Multiplication

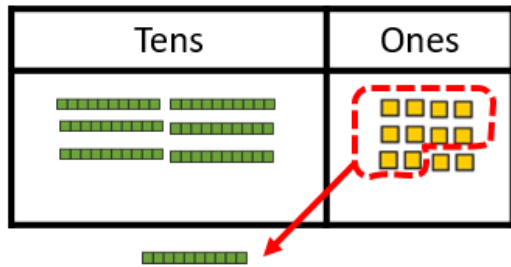
Year 3

NC Statement:

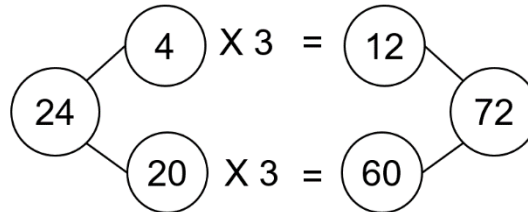
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

3LS26 Step 4: Short multiplication with regrouping of ones into tens only

Concrete



Pictorial - Jottings



Abstract - Written symbolic

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ \hline 1 \end{array}$$

$$24 \times 3 = 72$$

Abstract - Speaking frame

... groups of ... ones is ... ones.
I can regroup the ... ones into ... ten(s) and ... one(s).
 ... groups of ... tens is ... tens.
 ... ten(s) added to ... is ...
 The product of ... and ... is ...

Notes:

Pupils have already met the distributive law (3LS18) and rehearsed multiplying by ten (3LS25).

The focus of this step is to support pupils in making the connection between informal distributive approach and the formal layout.

Speaking frame note:

“3 groups of 4 ones is 12 ones. I can regroup the 12 ones into 1 ten and 2 ones. 3 groups of 2 tens is 6 tens. 1 ten added to 6 tens is 7 tens. The product of 24 x 3 is 72.”

Pupils should be encouraged to consider whether italicised language in the speaking frame is required in the calculation.

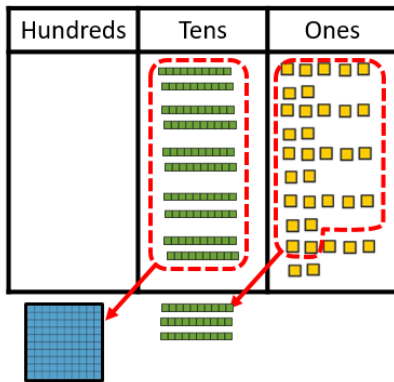
Year 3

NC Statement:

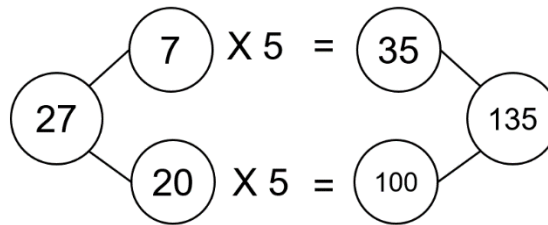
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

3LS26 Step 5: Short multiplication with regrouping of ones and tens

Concrete



Pictorial - Jottings



Abstract - Written symbolic

$$\begin{array}{r} 27 \\ \times 5 \\ \hline 135 \\ \hline 3 \end{array}$$

$$27 \times 5 = 135$$

Abstract - Speaking frame

... groups of ... ones is ... ones.
 I can regroup the ... ones into ... **ten(s)** and ... one(s).
 ... groups of ... tens is ... tens.
 ... ten(s) added to ... ten(s) is ...
 I can regroup the ... tens into ... **hundred(s)** and ... ten(s)
 The product of ... and ... is ...

Notes:

At this stage, the pictorial representation is being used as a checking point to ensure pupils answer accurately. This allows focused attention on understanding the abstract recording.

Speaking frame note:

"5 groups of 7 ones is 35 ones. I can regroup the 35 ones into 3 tens and 5 ones. 5 groups of 2 tens is 10 tens. 3 tens added to 10 tens is 13 tens. I can regroup the 13 tens into 1 hundred and 3 tens. The product of 27 x 5 is 135."

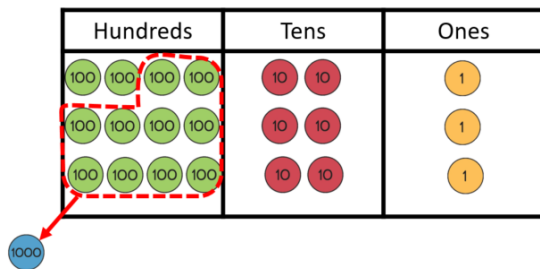
Year 4

NC Statement:

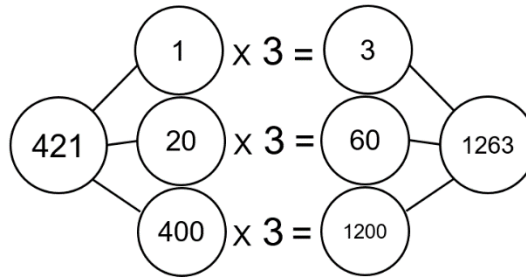
Multiply 2-digit and 3-digit numbers by a one-digit number using formal written layout (short multiplication)

4LS24 Step 5: Short multiplication with regrouping in one column (regrouping generates a new column)

Concrete



Pictorial - Jottings



Abstract - Written symbolic

$$\begin{array}{r}
 421 \\
 \times \quad 3 \\
 \hline
 1263
 \end{array}$$

$$421 \times 3 = 1263$$

Abstract - Speaking frame

... groups of ... ones is ... ones. (*Do I need to regroup?*)
 ... groups of ... tens is ... tens. (*Do I need to regroup?*)
 ... groups of ... hundreds is ... hundreds. (*Do I need to regroup?*)
 (... hundreds can be regrouped to ... thousands and ... hundreds)
 The product of ... and ... is ...

Notes:

At this stage, the pictorial representation is being used as a checking point to ensure that pupils answer accurately. This allows focused attention on understanding the abstract recording.

Pupils should be encouraged to consider whether the italicised language in the speaking frame is required in the calculation.

Multiplication

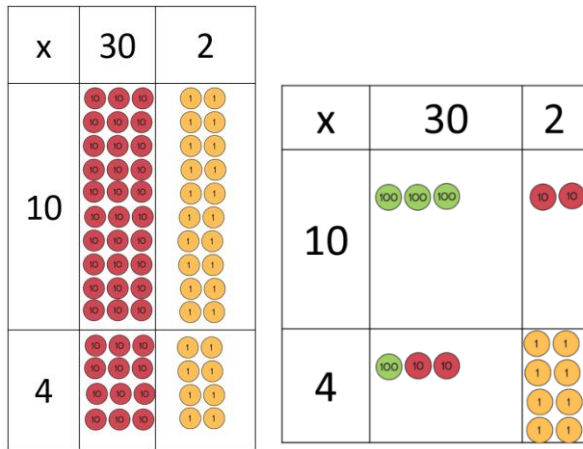
Year 5

NC Statement:

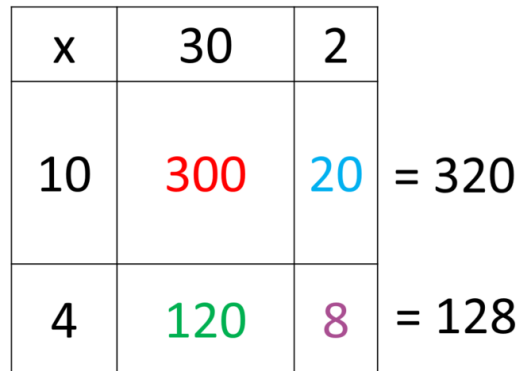
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

5LS11 Step 2: Using grid method and expanded vertical multiplication to solve a 2-digit number by 2-digit number multiplication

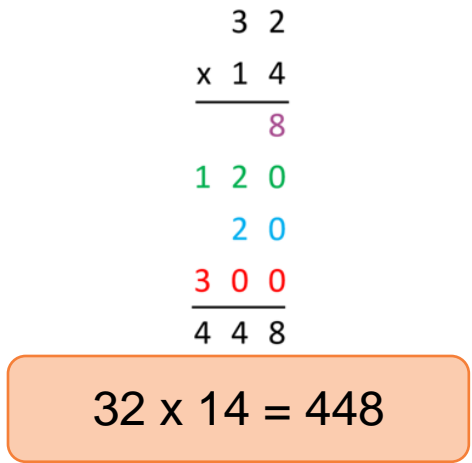
Concrete



Pictorial - Jottings



Abstract - Written symbolic



Abstract - Speaking frame

First, I need to consider the ones in the multiplier.
 ... groups of ... ones is **ones**.
 ... groups of ... tens is **tens**. (Do I need to regroup?)
 Then, tens in the multiplier.
 ... groups of ... ones is **ones**. (Do I need to regroup?)
 ... groups of ... tens is **tens**. (Do I need to regroup?)
 The total of all the partial products is
 The product of ... and ... is

Notes:

This is a transitional method towards long multiplication. Using the grid supports pupils in their thinking about multiplying by powers of ten and place value. Secure understanding of both of these concepts allow pupils to move to long multiplication more successfully.

Speaking frame hint: linking to what we know and correct place value. For example, 10 groups of 3 tens is 30 tens. This can be regrouped to 3 hundreds.

Multiplication

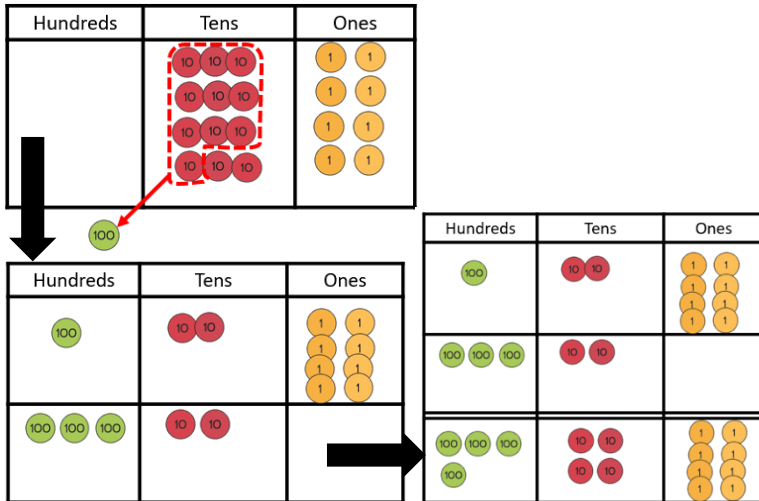
Year 5

NC Statement:

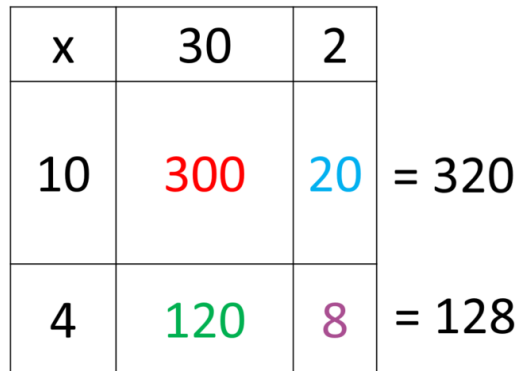
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

5LS11 Step 3: Long multiplication of a 2-digit number by 2-digit number (with simple regrouping)

Concrete



Pictorial - Jottings



Abstract - Written symbolic

$$\begin{array}{r} 32 \\ \times 14 \\ \hline 128 \\ 320 \\ \hline 448 \end{array}$$

$$32 \times 14 = 448$$

Abstract - Speaking frame

First, I need to consider the ones in the multiplier.
 ... groups of ... ones is **ones**. (Do I need to regroup?)
 ... groups of ... tens is **tens**. (Do I need to regroup?)
 Then, considering tens in the multiplier.
 ... groups of ... ones is **ones**. (Do I need to regroup?)
 ... groups of ... tens is **tens**. (Do I need to regroup?)
 The total of all the partial products is
 The product of ... and ... is

Notes:

Speaking frame hint: linking to what we know and correct place value.
 For example, 10 groups of 3 tens is 30 tens (linking to known fact 10×3). This can be regrouped to 3 hundreds.

Multiplication

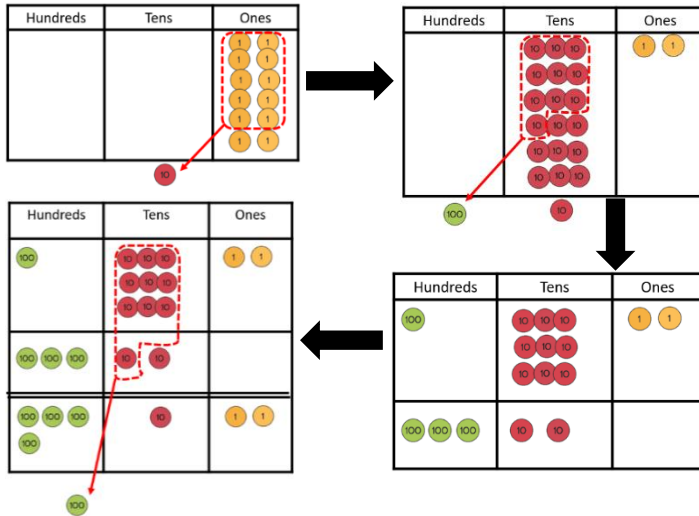
Year 5

NC Statement:

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

5LS11 Step 3: Long multiplication of a 2-digit number by a 2-digit number (focusing on regroup in first partial product)

Concrete



Pictorial

x	30	2	
10	300	20	= 320
6	180	12	= 192

Abstract - Written symbolic

$$\begin{array}{r}
 \overset{1}{3} 2 \\
 \times 16 \\
 \hline
 192 \\
 320 \\
 \hline
 512 \\
 \small{1}
 \end{array}$$

$$32 \times 16 = 512$$

Abstract - Speaking frame

First, I need to consider the ones in the multiplier.
 ... groups of ... ones is ... **ones**. (Do I need to regroup?)
 ... groups of ... tens is ... **tens**. (Any regroup to add? Do I need to regroup?)
 Then, considering tens in the multiplier.
 ... groups of ... ones is ... **ones**. (Do I need to regroup?)
 ... groups of ... tens is ... **tens**. (Do I need to regroup?)
 The total of all the partial products is
 The product of ... and ... is

Notes:

Speaking frame hint: linking to what we know and correct place value.

For example, 6 groups of 3 tens is 18 tens (linking to known fact $6 \times 3 = 18$). This can be regrouped to 1 hundred and 8 tens.

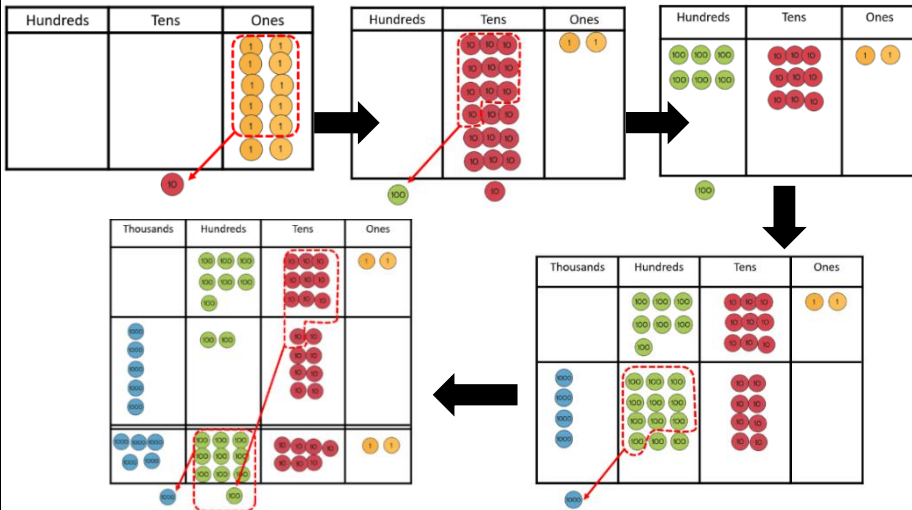
Year 5

NC Statement:

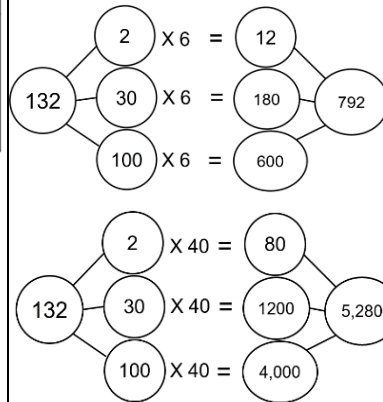
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

5LS11 Step 3: Long multiplication of a 2-digit number by 2-digit number (regrouping in first and second stage)

Concrete



Pictorial - Jottings



Abstract - Written symbolic

$$\begin{array}{r}
 132 \\
 \times 46 \\
 \hline
 792 \\
 5280 \\
 \hline
 6072
 \end{array}$$

$$132 \times 46 = 6,072$$

Abstract - Speaking frame

First, I need to consider the ones in the multiplier.
 ... groups of ... ones is ... ones. (Do I need to regroup?)
 ... groups of ... tens is ... tens. (Any regroups to add? Do I need to regroup?)
 Then, considering tens in the multiplier.
 ... groups of ... ones is ... ones. (Do I need to regroup?)
 ... groups of ... tens is ... tens. (Any regroups to add? Do I need to regroup?)
 The total of all the partial products is
 The product of ... and ... is

Notes:

Speaking frame hint: linking to what we know and correct place value.
 For example, 6 groups of 3 tens is 18 tens (linking to known fact $6 \times 3 = 18$). This can be regrouped to 1 hundred and 8 tens.

Multiplication

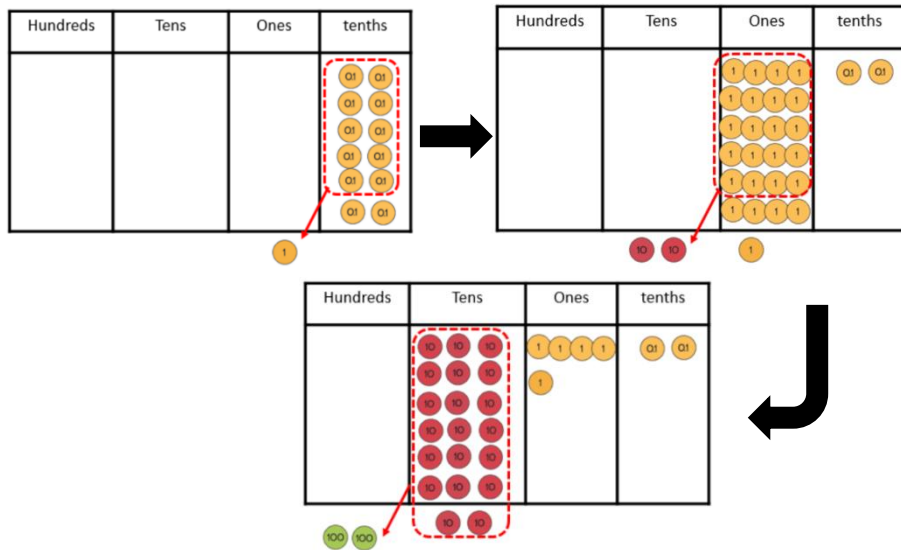
Year 6

NC Statement:

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

6LS6 Step 5: Long multiplication involving numbers with up to 2 decimal places multiplied by a 1-digit number

Concrete



Pictorial - Jottings

Jottings: multiples of tricky multipliers

- 6
- 12
- 18
- 24
- 30
- 36
- 42
- 48
- 54
- 60
- 66
- 72

Abstract - Written symbolic

$$\begin{array}{r} ^2 ^1 \\ 34.2 \\ \times 6 \\ \hline 205.2 \end{array}$$

$34.2 \times 6 = 205.2$

Abstract - Speaking frame

... groups of ... tenths is ... tenths. (Do I need to regroup?)
 ... groups of ... ones is ... ones. (Any regroup to add? Do I need to regroup?)
 ... groups of ... tens is ... tens. (Any regroup to add? Do I need to regroup?)
 The product of ... and ... is

Notes:

Speaking frame hint: linking to what we know and correct place value. For example, 6 groups of 3 tens is 18 tens (linking to known fact $6 \times 3 = 18$). This can be regrouped to 1 hundred and 8 tens.

DIVISION PROGRESSION

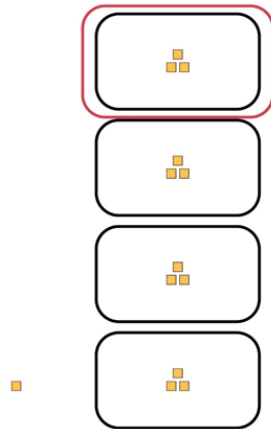
Year 3

NC Statement:

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

3LS30 Step 2: Introducing the long division method (sharing ones)

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} 3 \\ 4 \overline{) 13} \\ - 12 \\ \hline 1 \end{array}$$

$$13 \div 4 = 3 \text{ r } 1$$

Abstract - Speaking frame

I am sharing ... ones into ... equal groups.
 There are ... ones in each group.
 I have ... one(s) remaining.
 The quotient is ... with ... remainders.

Notes:

Pupils are introduced to the long division method for the first time in this sequence. Short division will not be introduced until pupils have understood all of the stages in this expanded form. In the calculation $96 \div 4$, for example, pupils often struggle to understand that 1 ten will be regrouped after 8 tens have been used in the 4 groups. This is hidden in short division but recorded in long division.

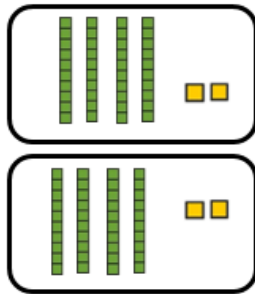
Year 3

NC Statement:

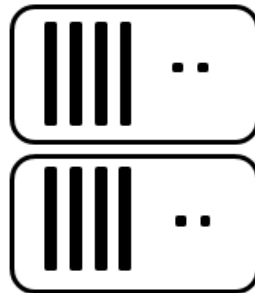
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

3LS30 Step 3: Long division of tens and ones with no regrouping (sharing structure)

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r}
 4 \quad 2 \\
 2 \overline{) 84} \\
 \underline{- 8} \quad 4 \\
 0 \quad 4 \\
 \underline{ 4} \\
 0
 \end{array}$$

$$84 \div 2 = 42$$

Abstract - Speaking frame

First, I am sharing ... tens into ... equal groups.
 There are ... tens in each group.
 I have ... ten(s) remaining.
 Then, I am sharing ... ones into ... equal groups.
 There are ... ones in each group.
 I have ... one(s) remaining.
 The quotient is ... with ... remainders.

Notes:

This stage is to support pupils' understanding of the abstract notation. They learn to record how many tens are in each group, if there are any tens remaining and what the arrow means.

Speaking frame note: "First, I am sharing 8 tens into 2 equal groups. There are 4 tens in each group. I have zero tens remaining. Then, I am sharing 4 ones into 2 equal groups. There are 2 ones in each group. I have zero ones remaining. The quotient is 42 with no remainders."

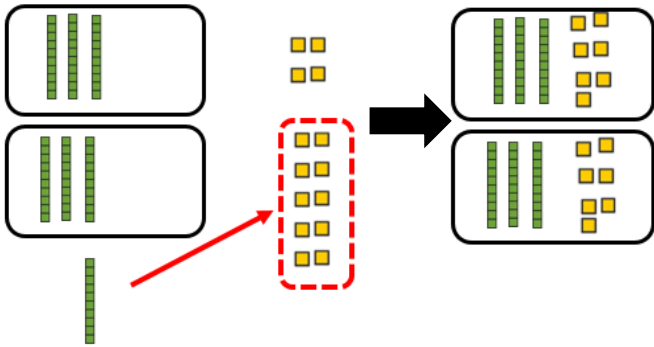
Year 3

NC Statement:

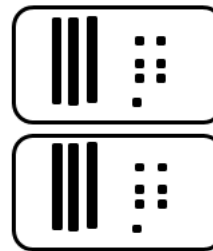
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

3LS30 Step 4: Long division of tens and ones with regrouping (sharing structure)

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r}
 37 \\
 2 \overline{) 74} \\
 \underline{- 6} \\
 14 \\
 \underline{- 14} \\
 0
 \end{array}$$

$$74 \div 2 = 37$$

Abstract - Speaking frame

First, I am sharing ... tens into ... equal groups.
 There are ... tens in each group.
 I have ... ten(s) remaining.
 I need to regroup the remaining ... ten(s) into ... ones.
 I now have ... ones in total.
 Then, I am sharing ... ones into ... equal groups.
 There are ... ones in each group.
 I have ... one(s) remaining.
 The quotient is ... with ... remainders.

Notes:

This is a crucial stage as it demonstrates the regrouping of the remaining tens for ones and how this is recorded abstractly.

Speaking frame note: "... I have 1 ten remaining. I need to regroup the remaining 1 ten into 10 ones. I now have 14 ones in total..."

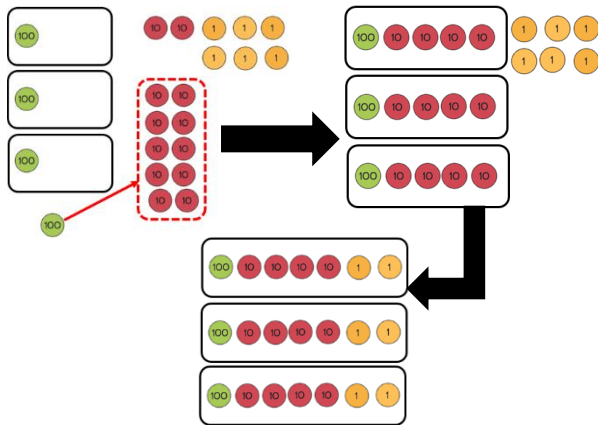
Year 4

NC Statement:

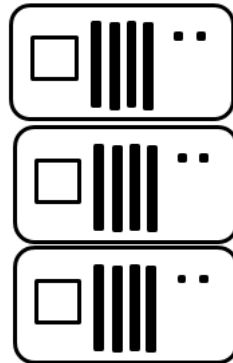
Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers (non-statutory guidance)

4LS25 Step 2: Long division with regrouping hundreds into tens (sharing structure)

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r}
 142 \\
 3 \overline{) 426} \\
 \underline{- 3} \\
 12 \\
 \underline{- 12} \\
 06 \\
 \underline{- 06} \\
 0
 \end{array}$$

$$426 \div 3 = 142$$

Abstract - Speaking frame

First, I am sharing ... hundreds into ... equal groups.
 There are ... hundreds in each group.
 I have ... hundred(s) remaining.
 I need to regroup the remaining ... hundreds into ... tens.
 I now have ... tens in total.
 Next, I am sharing ... tens into .. equal groups.

Notes:

Pupils revisit long division with no regrouping in 4LS25 step 1. This is to ensure that they understand the abstract recording of long division.

Speaking frame note: This stage is an extension to the previous speaking frame – focusing on the hundreds regroup.

Division

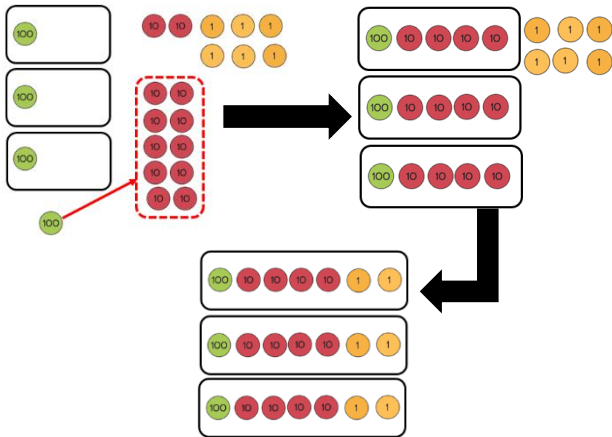
Year 4

NC Statement:

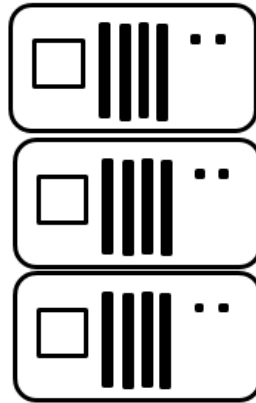
Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers (non-statutory guidance)

4LS25 Step 4: Short division (sharing structure)

Concrete



Pictorial



Abstract - Written symbolic

$$\begin{array}{r} 142 \\ 3 \overline{) 426} \\ \underline{3} \\ 12 \\ \underline{9} \\ 36 \\ \underline{36} \\ 0 \end{array}$$

$$426 \div 3 = 142$$

Abstract - Speaking frame

First, I am sharing ... hundreds into ... equal groups.
 There are ... hundreds in each group.
 I have ... hundred(s) remaining.
 I need to regroup the remaining ... hundreds into ... **tens**.
 I now have ... tens in total.
 Next, I am sharing ... tens into .. equal groups.

Notes:

In this stage, pupils learn that the thinking processes for long and short division are the same – it is only the abstract written that is different. It is important that pupils are able to link this to the long division format and can explain the compaction.

Speaking frame note: This stage is an extension to the previous speaking frame – focusing on the hundreds regroup.

Division

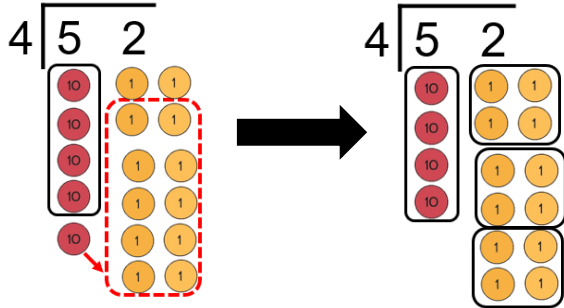
Year 5

NC Statement:

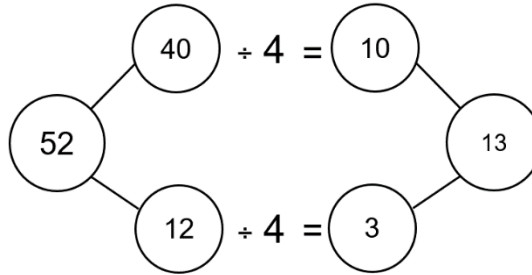
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

5LS12 Step 2: Sharing and grouping (grouping structure)

Concrete



Pictorial - Jottings



Abstract - Written symbolic

$$\begin{array}{r} 13 \\ 4 \overline{) 52} \end{array}$$

$$52 \div 4 = 13$$

Abstract - Speaking frame

I want to know how many groups of ... are in
 How many groups of ... tens are in ... tens without regrouping?
 I can make ... group(s) of ... tens. There is/are ... ten(s) remaining.
 I need to regroup the ... tens into ... ones.
 I now have ... ones.
 How many groups of ... ones are in ... ones, without regrouping?
 I can make ... group(s) of ... ones. There is/are ... one(s) remaining.
 There are ... groups of ... in ... with ... remainders.

Notes:

Pupils are encouraged to progress to a grouping model of division. This is in preparation for 2-digit divisors and understanding fractions expressed as part of the quotient.

Pupils should explore with simple division calculations to ensure that they understand the shift in structure.

Speaking frame note: In this example, the speaking frame would be completed like this: "How many groups of 3 tens are in 4 tens, without regrouping?"

This is to ensure that accurate place value and magnitude is maintained.

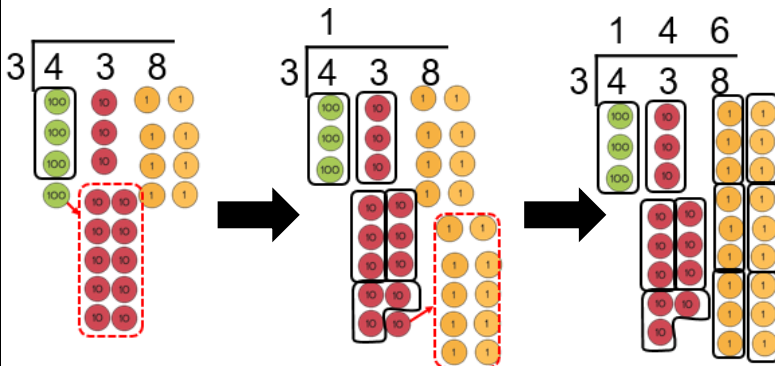
Year 5

NC Statement:

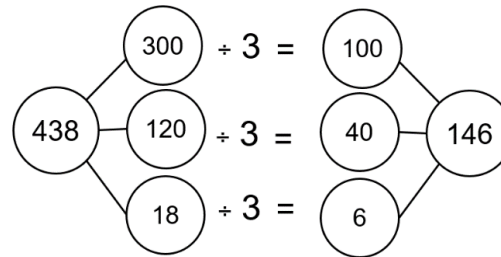
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

5LS12 Step 3: Short division for numbers up to 4-digits (grouping structure)

Concrete



Pictorial - Jottings



Abstract - Written symbolic

$$\begin{array}{r} 146 \\ 3 \overline{) 438} \end{array}$$

$$438 \div 3 = 146$$

Abstract - Speaking frame

I want to know how many groups of ... are in
 How many groups of ... hundreds are in ... hundreds, without regrouping?
 I can make ... group(s) of ...hundreds. There is/are ... hundred(s) remaining.
 I need to regroup the ... hundreds into ...tens.

Notes:

Speaking frame note: This is an extension to the previous speaking frame. In this example, the speaking frame would be completed like this:
 "How many groups of 3 hundreds are in 4 hundreds, without regrouping?"
 This is to ensure that accurate place value and magnitude is maintained.

Division

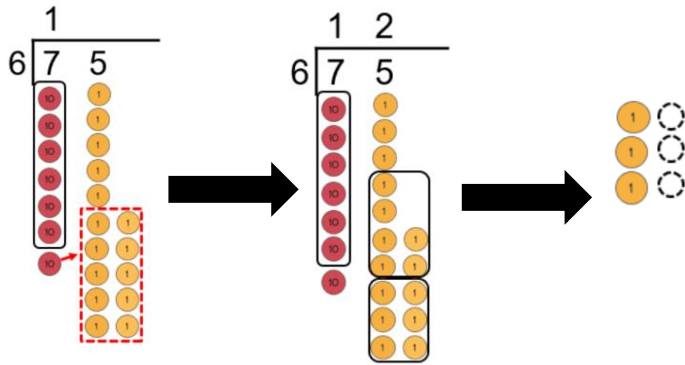
Year 5

NC Statement:

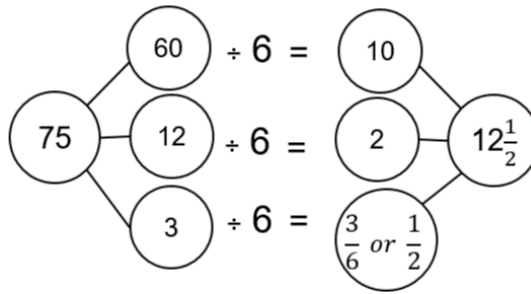
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

5LS12 Step 4: Expressing quotients with fractions

Concrete



Pictorial



Abstract - Written symbolic

$$6 \overline{) 75} \begin{matrix} 12 \\ \underline{12} \\ 15 \end{matrix} \frac{1}{2}$$

$$75 \div 6 = 12 \frac{1}{2}$$

Abstract - Speaking frame

I have a remainder of

This is ... (remainder) out of ... (divisor) which I need for another group.

This can be written as a fraction $\frac{\square}{\square}$.

This can be simplified to $\frac{\square}{\square}$.

Notes:

Speaking frame note: This is an extension to the previous speaking frame (5LS12 Step 2). In this example the speaking frame would be completed like this:

"I have a remainder of 3.

This is 3 out of 6 which I need for another group.

This can be written as a fraction $\frac{3}{6}$.

This can be simplified to $\frac{1}{2}$."

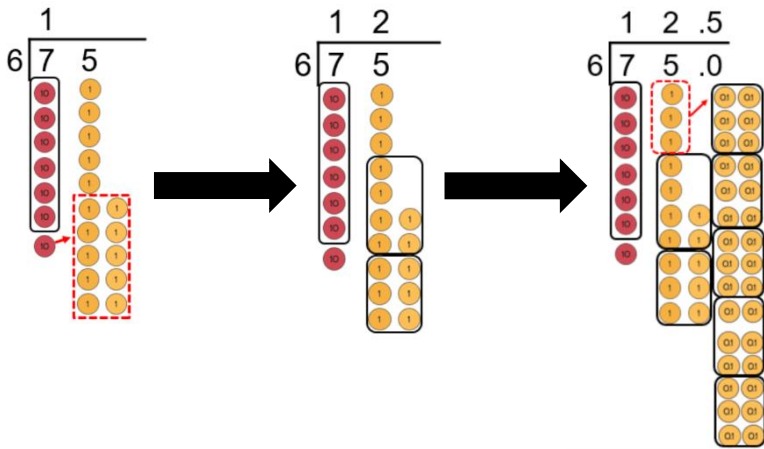
Year 5

NC Statement:

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

5LS12 Step 5: Expressing quotients with decimals

Concrete



Pictorial - Jottings

Jottings: multiples of the divisor

- 6
- 12
- 18
- 24
- 30
- 36
- 42
- 48
- 54
- 60

Abstract - Written symbolic

$$\begin{array}{r} 12.5 \\ 6 \overline{) 75.0} \\ \underline{6} \\ 15 \\ \underline{12} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

$$75 \div 6 = 12.5$$

Abstract - Speaking frame

I have a remainder of
 I need to regroup the ... ones into ... tenths.
 How many groups of ... tenths are in ... tenths, without regrouping?
 I can make ... group(s) of ... tenths.

 There are ... groups of ... in

Notes:

Speaking frame note: This is an extension to the previous speaking frame (5LS12 Step 2). In this example, the speaking frame would be completed like this:

*"I have a remainder of 3.
 I need to regroup the 3 ones into 30 tenths.
 How many groups of 6 tenths are in 30 tenths, without regrouping?
 I can make 5 groups of 6 tenths.
 There are 12.5 groups of 6 in 75."*

Division

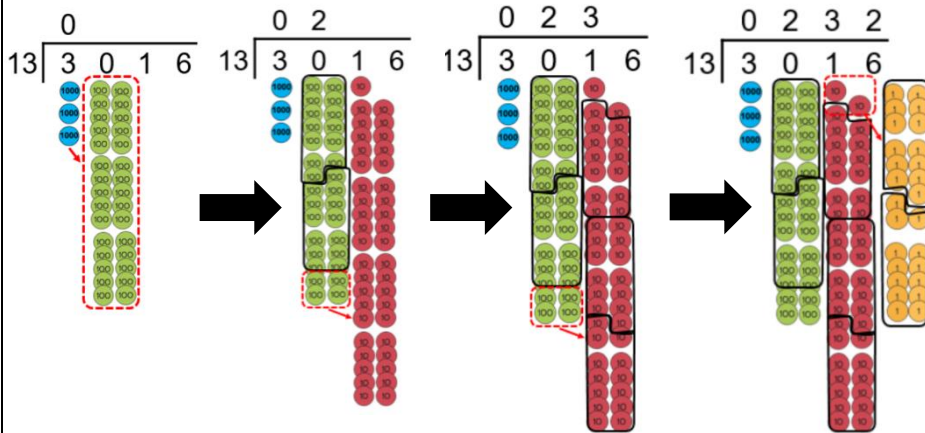
Year 6

NC Statement:

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

6LS17 Step 2: Long division for numbers up to 4 digits

Concrete



Pictorial - Jottings

Jottings: multiples of the divisor

- 13
- 26
- 39
- 52
- 65
- 78
- 91
- 104

Abstract - Written symbolic

$$\begin{array}{r}
 2 \\
 13 \overline{) 3016} \\
 \underline{- 0} \\
 30 \\
 \underline{- 26} \\
 41 \\
 \underline{- 39} \\
 26 \\
 \underline{- 26} \\
 0
 \end{array}$$

$$3016 \div 13 = 232$$

Abstract - Speaking frame

I want to know how many groups of ... are in
 How many groups of ... thousand are in ...thousand, without regrouping?
 I can make ... group(s) of ...thousand. There is/are ... thousand(s) remaining.
 I need to regroup the ... thousand(s) into ...hundreds.

Notes:

The structure of long division was first introduced in 3LS30, then revisited and extended in both years 4 and 5. It was revised in Step 1 of this sequence.

Jottings are used to scaffold to derived related division facts.

Speaking frame note: This is an extension to the previous speaking frame (5LS12 Step 2). In this example, the speaking frame would be completed like this:

“How many groups of 13 thousands are in 3 thousand, without regrouping?” I can make zero groups of 13 thousand. There are 3 thousand remaining. I need to regroup the 3 thousands into 30 hundreds.”

These additional examples show only jottings, completed speaking frames and abstract recording. This complexity of calculation should only be introduced to pupils once they are confident in the conceptual pathway and can explain the abstract recording with reference to the concrete and pictorial models.

Additional Year 6 examples

Year 6

NC Statement:

Multiply multi-digit numbers of up to 4-digits by a two-digit whole number using the formal written method of long multiplication

6LS6 Step 3 (revision): Long multiplication; up to 4-digit by 2-digit

Abstract speaking frame

First, I need to consider the ones in the multiplier.

7 groups of 6 ones is 42 ones.

I need to regroup into 4 tens and 2 ones.

7 groups of 3 tens is 21 tens.

I need to add the regrouped 4 tens. I now have 25 tens.

I need to regroup into 2 hundreds and 5 tens.

7 groups of 8 hundreds is 56 hundreds.

I need to add the regrouped 2 hundreds. I now have 58 hundreds. I can regroup this into 5 thousands and 8 hundreds.

Then, considering the tens in the multiplier.

20 groups of 6 ones is 120 ones.

I need to regroup into 1 hundred and 2 tens.

20 groups of 3 tens is 6 hundreds.

I need to add the regrouped 1 hundred. I now have 7 hundreds.

20 groups of 8 hundred is 16 thousand. There are no regroupings to add.

The total of the two partial products is 22, 572.

The product of 836 and 27 is 22, 572.

Pictorial - Jottings

Jottings: multiples of tricky multipliers

7
14
21
28
35
42
49
56
63
70
77
84

Abstract - Written symbolic

$$\begin{array}{r}
 \begin{array}{r}
 \overset{1}{2} \overset{4}{4} \\
 836 \\
 \times 27 \\
 \hline
 5852 \\
 16720 \\
 \hline
 22572 \\
 11
 \end{array}
 \end{array}$$

$$836 \times 27 = 22,572$$

Additional Year 6 examples

NC Statement:

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

6LS17 Step 4: Expressing quotients with fractions

Abstract speaking frame

I have a remainder of 9.
 This is 9 out of the 15 which I need for another group.
 This can be written as a fraction $\frac{9}{15}$.
 This can be simplified to $\frac{3}{5}$.
 There are $37\frac{3}{5}$ in each of the 15 groups.

Pictorial - Jottings

Jottings: multiples of the divisor

- 15
- 30
- 45
- 60
- 75
- 90
- 105
- 120
- 135
- 150

Abstract - Written symbolic

$$\begin{array}{r}
 0 \quad 3 \quad 7 \quad \frac{3}{5} \\
 15 \overline{) 564} \\
 \underline{- 0} \quad \downarrow \quad \downarrow \\
 \quad 5 \quad 6 \\
 \underline{- 4} \quad 5 \\
 \quad \quad 1 \quad 1 \quad 4 \\
 \underline{- 1} \quad 0 \quad 5 \\
 \quad \quad \quad \quad \quad 9
 \end{array}$$

$$\frac{9}{15} = \frac{3}{5}$$

$$564 \div 15 = 37 \frac{3}{5}$$

Year 6

NC Statement:

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

6LS17 Step 5: Expressing quotients with decimals

Abstract speaking frame

I have a remainder of 9.
 I need to regroup the 9 ones into
 90 tenths.
 How many groups of 15 tenths are
 in 90 tenths, without regrouping?
 I can make 6 groups of 15 tenths.
 There is nothing remaining.
 There are 37.6 groups of 15 in 564.

Pictorial - Jottings

Jottings: multiples of the divisor

- 15
- 30
- 45
- 60
- 75
- 90
- 105
- 120
- 135
- 150

Abstract - Written symbolic

$$\begin{array}{r}
 \\
 15 \overline{) 564.0} \\
 \underline{- 0} \\
 5 \\
 \underline{- 4} \\
 1 \\
 \underline{- 1} \\
 \\
 9 \\
 \underline{- 9} \\
 0
 \end{array}$$

$$564 \div 15 = 37.6$$